JANATA MAHAVIDYALAYA, CHANDRAPUR

DEPARTMENT OF MATHEMATICS

SEM II (PAPER I)

Topic – Partial Diffrential Equation

By

DR SR GOMKAR

Partial Differential Equation

Definition :-

An equation containing partial derivatives with respect to x and y is called as partial differential equation.

If Z = f(x, y) is a function then,

$$P= \frac{\partial z}{\partial x}, \quad q= \frac{\partial z}{\partial y}, \quad r=\frac{\partial^2 z}{\partial x^2}, \quad s=\frac{\partial^2 z}{\partial x \partial y}, \quad t= \frac{\partial^2 z}{\partial y^2}$$

The equation of the type Pdx + Qdy + Rdz = 0 is called as partial differential equation.

Charpits Method :-

Consider non linear partial differential equation of first order f (x,y,z,p,q) =0____ ①

Then there exist another linear partial differential equation g (x,y,z,p,q) =0_____ ②

Such that equation ① and ② are compatible.

We know that

$$\frac{\mathrm{dx}}{\mathrm{fp}} = \frac{dy}{fq} = \frac{dz}{p.fp + q.fq} = \frac{dp}{-fx - pfz} = \frac{dq}{-fy - qfz}$$

This is the Charpits Equation.

Solving for p or q in the other form given equation and solution obtained form,

$Dz=pdx+qdy$

Solution is also called as complete integral.

Special Type :- Case–1

f(p,q) =0

i.e x,y,z are absent

$\Rightarrow$fx = fy = fz = 0

Then by Charpits Method,

$$\frac{\mathrm{dx}}{\mathrm{fp}} = \frac{dy}{fq} = \frac{dz}{p.fp + q.fq} = \frac{dp}{-fx - pfz} = \frac{dq}{-fy - qfz}$$

$\Rightarrow$dp=0 or dq=0

Integrating,

p=constant=a

Putting the value of p in given equation,

f(a,q) = 0

$\Rightarrow$q=?

Substitute the value of p and q in the solution

p.dx+q.dy=0

This is the complete integral.

Que:-Solve the partial Differential Equation by Charpits Method.

(1) p+q=pq ______ ①

Let, $f=p+q-pq=0$

As x,y,z are absent then,

$fx=fy=fz=0$

Then by Charpits Method;

$$\frac{\mathrm{dx}}{\mathrm{fp}}=\frac{dy}{fq}=\frac{dz}{p.fp+q.fq}=\frac{dp}{-fx-pfz}=\frac{dq}{-fy-qfz}$$

$\Rightarrow dp=0$ or $dq=0$

Integrating,

p= constant = a

Put in equation 1

$a+q=aq$

$a=aq-q$

$a=(a-1)q$

$q=\frac{a}{(a-1)}$

For solution,

$dz=p.dx+q.dq$

$dz=a.dx+\frac{a}{(a-1)}\ .\ dy$

Integrating

$$\boxed{Z=a.x+\frac{a}{(a-1)}.y+b}$$

This is the complete integral.

(2) $q=3p^2$

$q=3p^2$ ________ ①

$f=3p^2-q=0$

As x,y,z are absent

$fx=fy=fz=0$

By Charpits Method,

$$\frac{dx}{fp}=\frac{dy}{fq}=\frac{dz}{p.fp+q.fq}=\frac{dp}{-fx-pfz}=\frac{dq}{-fy-qfz}$$

$\Rightarrow dp=0$ or $dq=0$

Integrating,

P=constant=a

Put the value of p in Equation 1

$q=3a^2$

Substitute the value of p and q in solution

$dz=p.dx+q.dy$

$dz=a.dx+3a^2.dy$

Integrating,

$$\boxed{Z=a.x+3a^2.y+b}$$

This is the complete Integral.

THANK YOU