Janata Mahavidyalaya, Chandrapur

DEPARTMENT OF Mathematics

SEM – VI (PAPER I)

Topic: Relativity

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Question: Show that in nature , no signal moves with a velocity greater than the velocity of light.

Solution: Assume that we can emit signals with a velocity greater than the signal of light.

Let a signal can be sent at the time t=t'=0[when the two inertial systems S and S' coincide] from the common origins O,O' along the negative X' axis with a constant velocity u'>c relative to S'.

Let it arrive at the point P at $t_1'>0$.

If $O'P=x_p'$

Then $x_p'=-u' t_1' \ldots\ldots(1)$

In S frame the signal reaches P where $OP=x_p$ at time t_1.

By SLT, we have ,

$x_p= \alpha(x_p'+vt_1')$ and $t_1=\alpha (t_1'+\frac{v}{c2}x_p')$

$\Rightarrow$ $x_p= \alpha (-u' t_1'+vt_1')$

$x_p= \alpha t_1'(u'-v)$,by equation (1) .....(2)

Now, $t_1'>0$, $u'>c>v$ and hence $u'-v>0$

Then ,equation (2) implies that , $x_p<0$.

Also, $t_1= \alpha(t_1'-\frac{v}{c^2}u' t_1')$, by (1)

$$t_1 =\alpha t_1'(1-\frac{vu'}{c^2}) \quad \ldots\ldots(3)$$

Immediately on arrival at P, the signal be sent back to O with a velocity w>c relative to S .

Let x be the position of the signal at time t.

Then its path is given by,

$x-x_p=w(t-t_1)$ ……(4)

At the origin O , x=0 & $t=t_2$.

Then the equation (4) gives

1. $x_p=w(t_2-t_1)$

$$t_2-t_1=-\frac{x_p}{w}$$

or $$t_2=t_1-\frac{x_p}{w}=\alpha t_1(1-\frac{vu'}{c^2})+\frac{\alpha t_1'}{w}(u'-v)$$ by equation (2) and (3)

$$= \alpha t_1'(1-\frac{vu'}{c^2}+\frac{u'-v}{w})$$

$$=-\alpha t_1'\{(\frac{vu'}{c^2}-1)-\frac{u'-v}{w}\}$$

$t_2 = -\alpha t_1' \left(\frac{vu'}{c^2} - 1\right)\left(1 - \frac{u' - v}{w\left(\frac{vu'}{c^2} - 1\right)}\right)$ ……(5)

Choose u' and w such that $\frac{vu'}{c^2} > 1$ i.e. $u' > \frac{c^2}{v}$ and $w > \frac{u' - v}{w\left(\frac{vu'}{c^2} - 1\right)}$

This choice ensures that u'>c and w>c

$\frac{vu'}{c^2} > 1$ $\qquad$ $\frac{u' - v}{w\left(\frac{vu'}{c^2} - 1\right)} < 1$

$\frac{vu'}{c^2} - 1 > 0$ $\qquad$ $1 - \frac{u' - v}{w\left(\frac{vu'}{c^2} - 1\right)} > 0$

Then equation (5) implies $t_2 < 0$ which is absurd (unreasonable) .
Hence, the initial assumption is wrong.
Thus in nature no signal can move with a velocity greater than c relative to any inertial system.