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B.Sc. I Sem-I

Session- 2019-20

Parallel & Perpendicular Axis Theorem

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WHAT IS MOMENT OF INERTIA?

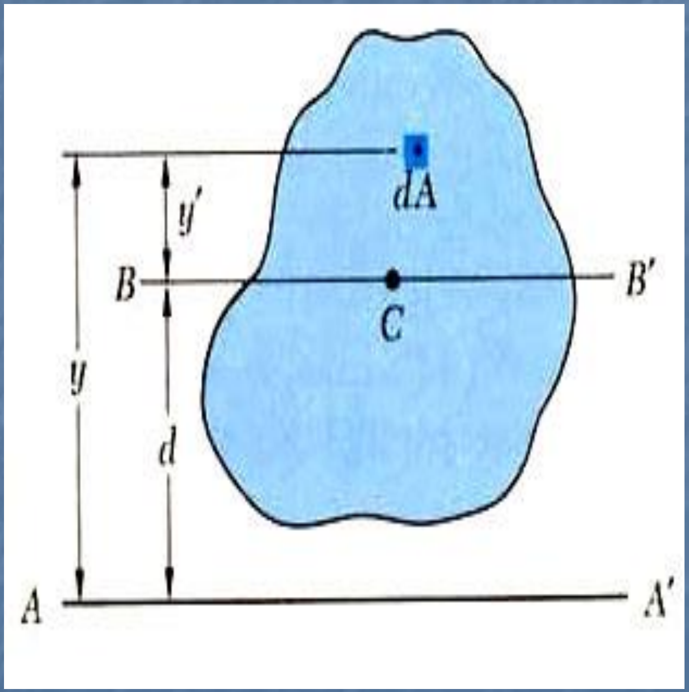
- IT IS THE MOMENT REQUIRED BY A SOLID BODY TO OVERCOME IT'S RESISTANCE TO ROTATION
- IT IS RESISTANCE OF BENDING MOMENT OF A BEAM
- IT IS THE SECOND MOMENT OF MASS (mr^2) OR SECOND MOMENT OF AREA (Ar^2)
- IT'S UNIT IS m^4 OR kgm^2

PERPENDICULAR AXIS THEOREM

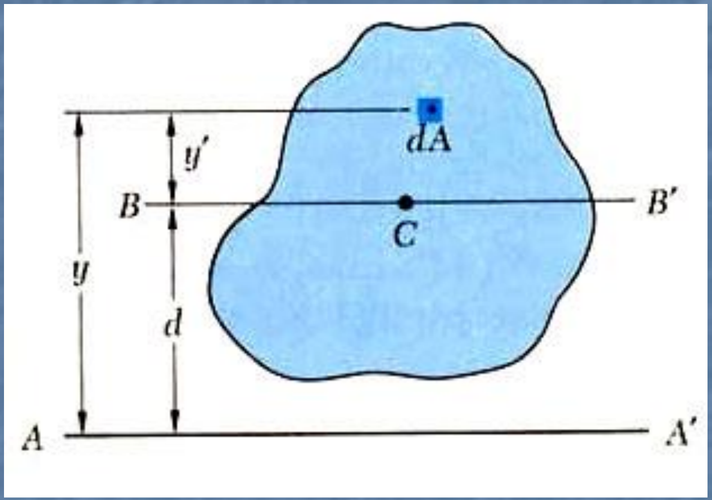
- The moment of inertia of a plane area about an axis normal to the plane is equal to the sum of the moments of inertia about any two mutually perpendicular axes lying in the plane and passing through the given axis.
  Moment of Inertia: $I_z = I_x + I_y$

PARALLEL AXIS THEOREM

- ***THE MOMENT OF AREA OF AN OBJECT ABOUT ANY AXIS PARALLEL TO THE CENTROIDAL AXIS IS THE SUM OF MI ABOUT IT'S CENTRODAL AXIS AND THE PRODUCT OF AREA WITH THE SQUARE OF DISTANCE OF CG FROM THE REF AXIS***
- $I_{XX} = I_G + Ad^2$
- A is the cross-sectional area.
  : is the perpendicuar distance between the centroidal axis and the parallel axis.



Moment of Inertia - Parallel Axis Theorem

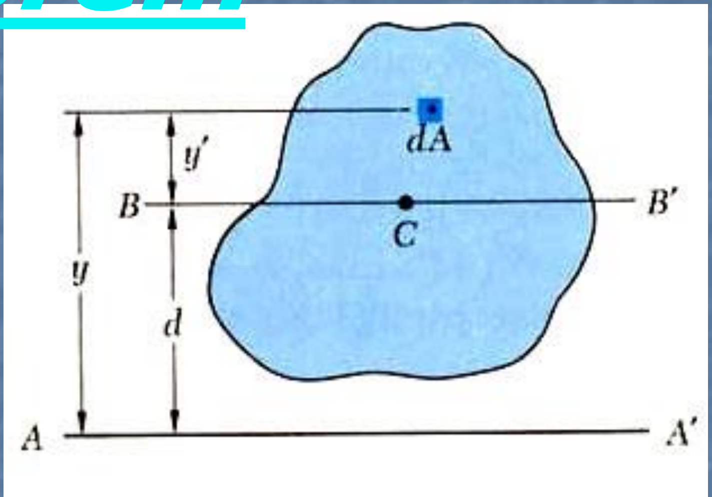


Parallel axis theorem:

Consider the moment of inertia Ix of an area A with respect to an axis AA'. Denote by y the distance from an element of area dA to AA'.

$$I_x = \int y^2 dA$$

Moment of Inertia - Parallel Axis Theorem



Consider an axis BB' parallel to AA' through the centroid C of the area, known as the centroidal axis. The equation of the moment inertia becomes

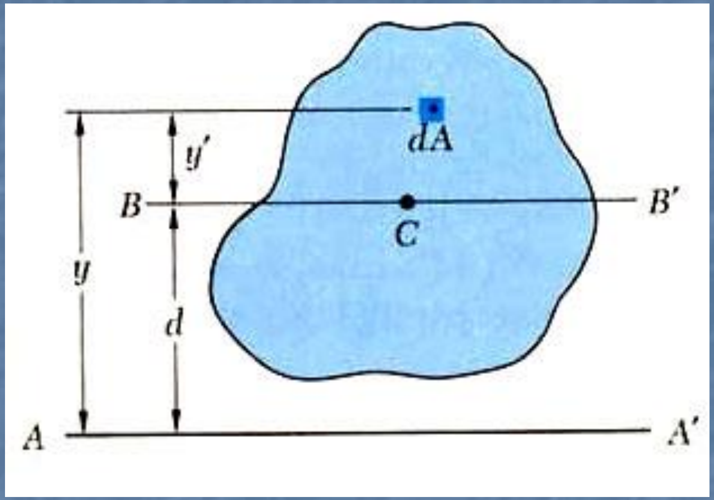
$$I_x = \int y^2 dA = \int (y' + d)^2 dA$$

$$= \int y'^2 dA + 2\int y' dA + d^2 \int dA$$

Moment of Inertia - Parallel Axis Theorem

The first integral is the moment of inertia about the centroid.

$$\overline{I}_x = \int y'^2 dA$$

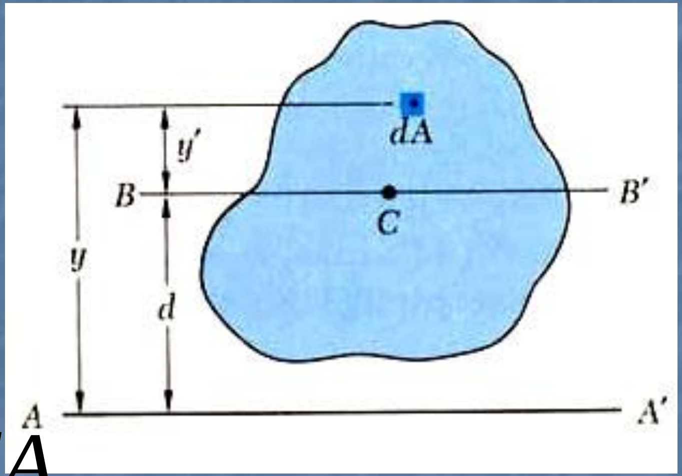


The second component is the first moment area about the centroid

$$\overline{y'} A = \int y' dA \Rightarrow \overline{y'} = 0$$

$$\Rightarrow \int y' dA = 0$$

Moment of Inertia - Parallel Axis Theorem



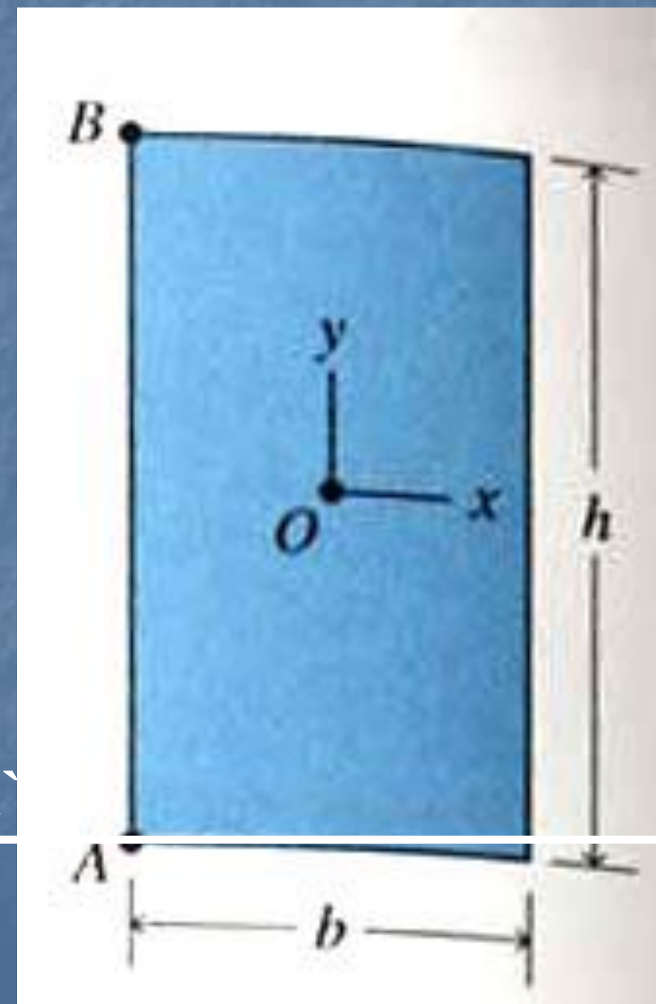
Modify the equation obtained with the parallel axis theorem.

$$I_x = \int y'^2 dA + 2\cancel{\int y' dA} + d^2 \int dA$$

$$= \overline{I}_x + d^2 A$$

Example – Moment of Inertia

Compute the moment of inertia in the x about the AA` plane.



AA`

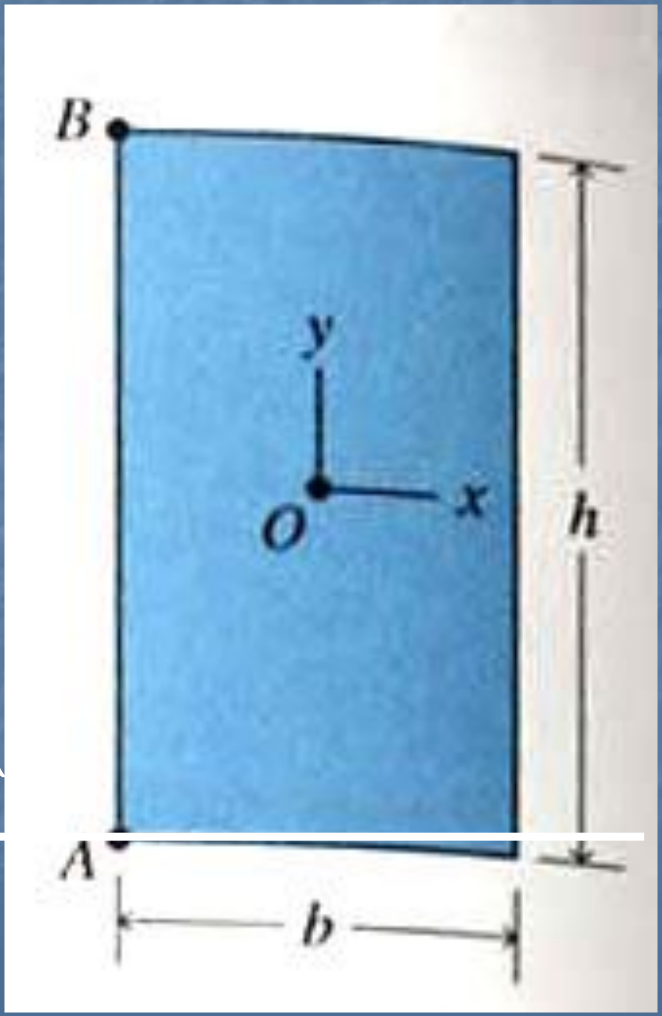
Example – Moment of Inertia

Compute the moment of inertia in the x about the AA` plane.

$$I_x = \int_{\text{Area}} y^2 dA = \int_0^h \int_0^b y^2 dx dy$$

$$= \left[b \frac{y^3}{3} \right]_0^h = \frac{bh^3}{3}$$

AA`



Thank You...