Janata Mahavidyalaya, Chandrapur

Department of Physics

B.Sc.-I Sem-I

Session- 2019-20

Concept of Moment of Inertia

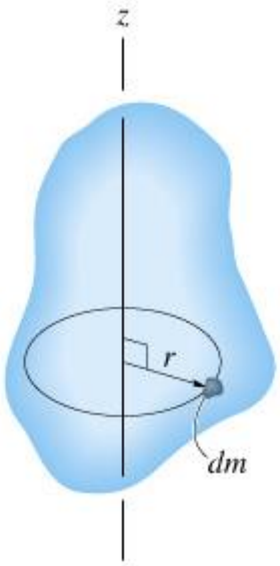
- Prof. G. P. Save

Dept. Of Physics

Moment of Inertia

The resistance of a body to changes in angular acceleration is described by the body's ***moment of inertia*** about the axis of rotation. By definition, the mass moment of inertia is

$$I = \int_m r^2 \, dm$$



Where *r* is the distance from the axis of rotation to the differential mass element *dm*.

In planar kinematics, the axis chosen for analysis is generally the one that passes through the center of gravity, and the mass moment of inertia through that axis is denoted I_G

If the body has a uniform density ρ, then dV = ρ *dm* and the mass moment of inertia can be written as

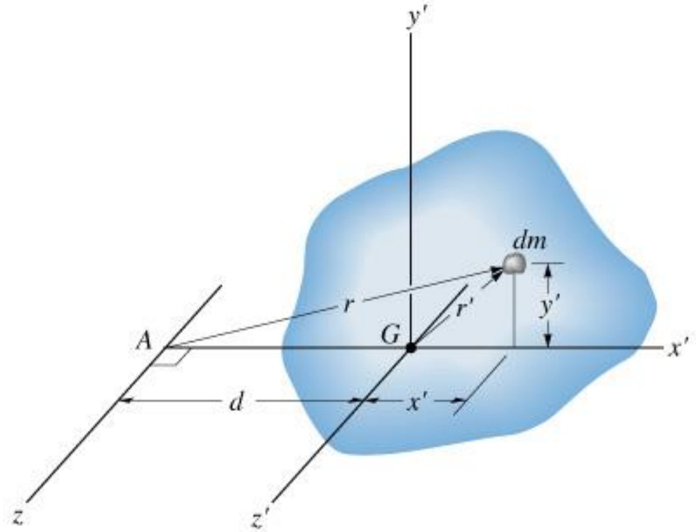
$$I = \int_m r^2 \, dm$$

$$I = \rho \int_V r^2 \, dV$$

Parallel-Axis Theorem

If the mass moment of inertia through the mass center is known, then the mass moment of inertia through any other, parallel, axis can be found.

$$I = I_G + md^2$$



Where:

I_G = moment of inertia about the axis passing through the mass center

I = moment of inertia about any parallel axis

m = total mass of the body

d = distance between the two parallel axes

Radius of Gyration

The mass moment of inertia can also be expressed as the radius of gyration, that distance at which the mass of the body can be concentrated to give the same moment of inertia about the axis through the mass center:

$$I_G = mk^2$$

$$k = \sqrt{\frac{I_G}{m}}$$

Composite Bodies

If a body is constructed of a number of simple shapes, its moment of inertia can be calculated by adding together all of the moments of inertia of the composite shapes about the desired axis:

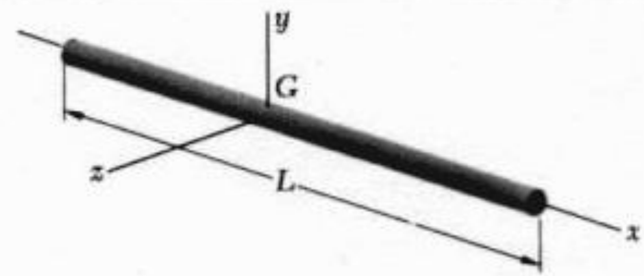
$$I = \sum (I_G + md^2)$$

Note that algebraic addition is necessary since it is possible to “subtract” a composite part from another, such as a hole from a plate.

See the following slides for a brief table of mass moments of inertia for various common shapes.

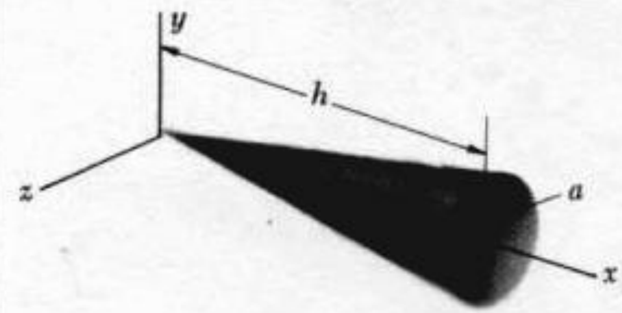
Slender rod

$I_y = I_z = \frac{1}{12}mL^2$



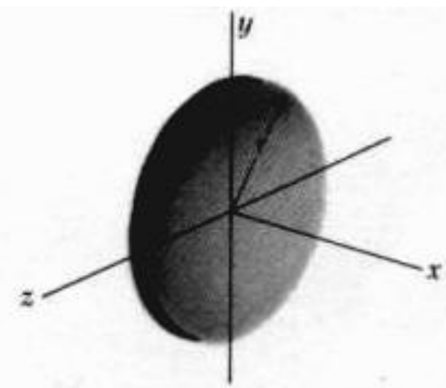
Circular cone

$I_x = \frac{3}{10}ma^2$
$I_y = I_z = \frac{3}{5}m(\frac{1}{4}a^2 + h^2)$



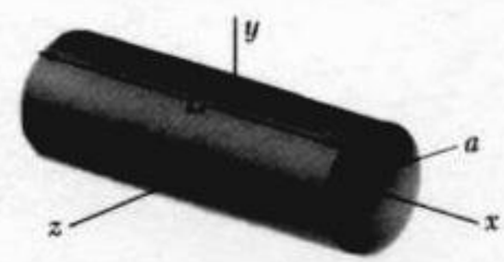
Thin disk

$I_x = \frac{1}{2}mr^2$
$I_y = I_z = \frac{1}{4}mr^2$



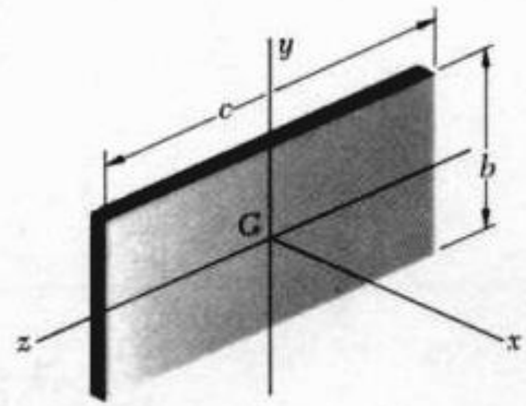
Circular cylinder

$I_x = \frac{1}{2}ma^2$
$I_y = I_z = \frac{1}{12}m(3a^2 + L^2)$



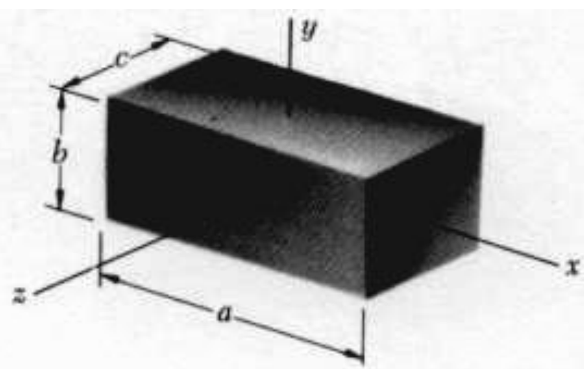
Thin rectangular plate

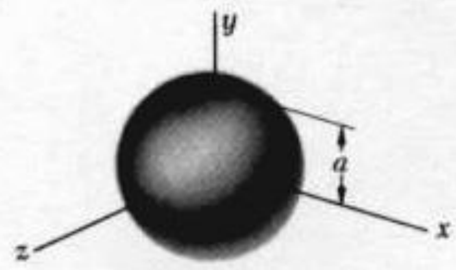
$I_x = \frac{1}{12}m(b^2 + c^2)$
$I_y = \frac{1}{12}mc^2$
$I_z = \frac{1}{12}mb^2$

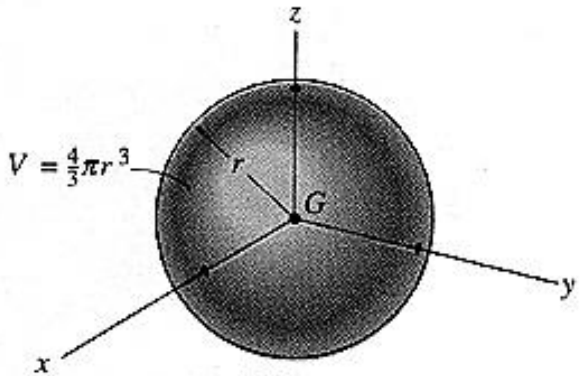


Rectangular prism

$I_x = \frac{1}{12}m(b^2 + c^2)$
$I_y = \frac{1}{12}m(c^2 + a^2)$
$I_z = \frac{1}{12}m(a^2 + b^2)$

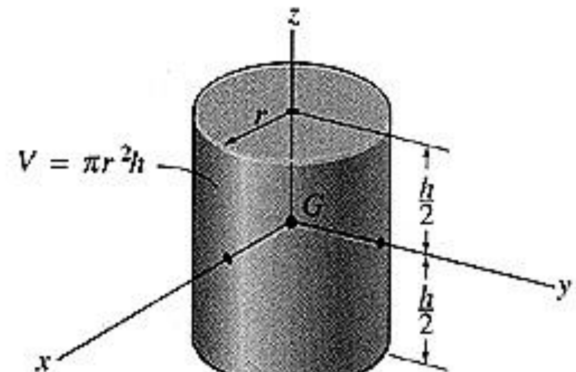


Sphere	
$I_x = I_y = I_z = \frac{2}{5}ma^2$	



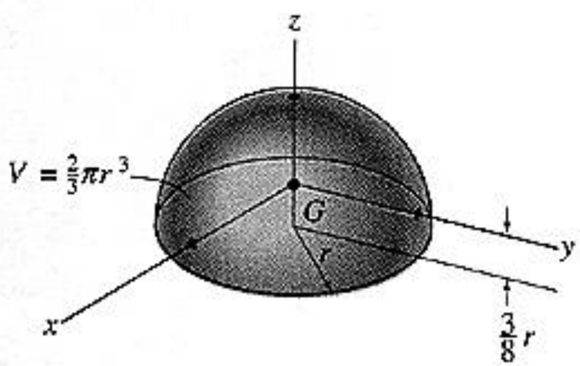
Sphere

$$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} m r^2$$



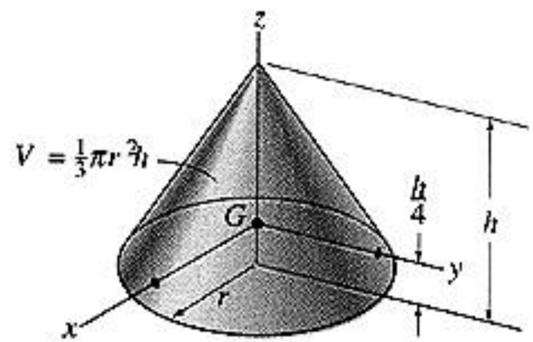
Cylinder

$$I_{xx} = I_{yy} = \frac{1}{12} m(3r^2 + h^2) \quad I_{zz} = \frac{1}{2} m r^2$$



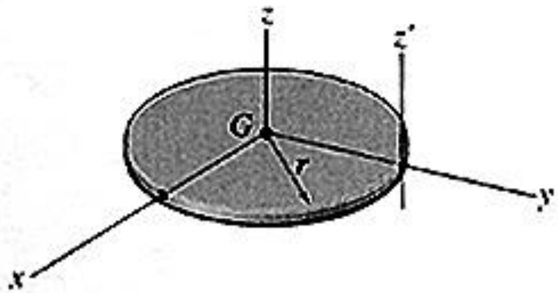
Hemisphere

$$I_{xx} = I_{yy} = 0.259 m r^2 \quad I_{zz} = \frac{2}{5} m r^2$$



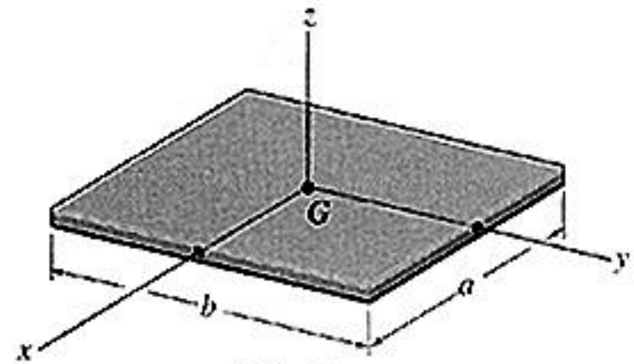
Cone

$$I_{xx} = I_{yy} = \frac{3}{80} m(4r^2 + h^2) \quad I_{zz} = \frac{3}{10} m r^2$$



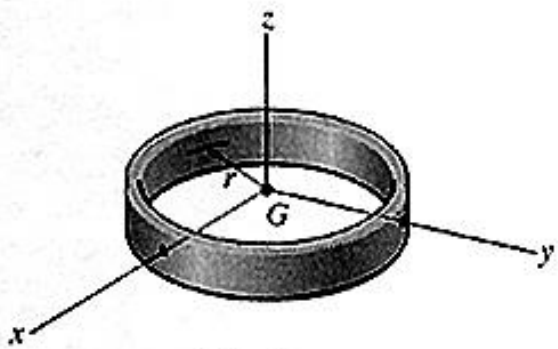
Thin circular disk

$I_{xx} = I_{yy} = \frac{1}{4} mr^2 \quad I_{zz} = \frac{1}{2} mr^2 \quad I_{z'z'} = \frac{3}{2} mr^2$



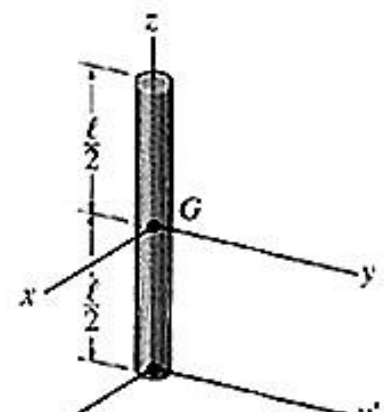
Thin plate

$I_{xx} = \frac{1}{12} mb^2 \quad I_{yy} = \frac{1}{12} ma^2 \quad I_{zz} = \frac{1}{12} m(a^2 + b^2)$



Thin ring

$I_{xx} = I_{yy} = \frac{1}{2} mr^2 \quad I_{zz} = mr^2$



Slender rod

$I_{xx} = I_{yy} = \frac{1}{12} m\ell^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3} m\ell^2 \quad I_{z'z'} = 0$

<u>Thank You</u>..............